NOTATION

τ, time; $\bar{T}$, temperature of melt; T_0, initial temperature; T_C, crystallization temperature of liquid; Ψ, stream function; φ, curl of velocity; ν, coefficient of kinematic viscosity; a, coefficient of thermal diffusivity; $\tilde{x}_0$, characteristic size of region; $\bar{e}_g$, unit vector of $0\eta_3$ axis; l_3, relative height of cavity; ε_i, relative width of liquid zone ($i = 1, 3$); ω_h, coordinate grid of region; h, distance between nodes of coordinate grid; A, time multiplier; $T_0 - T_C$, characteristic temperature difference; $\tilde{p} = \rho\tilde{u}_0^2$, characteristic pressure; $R = PrGr$, Rayleigh number; $\eta_i = x_i/\tilde{x}_0$, dimensionless coordinates ($i = 1, 3$); $\Theta = (T - T_C)/(T_0 - T_C)$, dimensionless temperature; $\pi = p/\tilde{p}$, dimensionless pressure; $Fo = \tau/\tilde{\tau}$, dimensionless time (Fourier number); $\tilde{u}_0 = \nu/\tilde{x}_0$, characteristic velocity; $\tilde{\tau} = \tilde{x}_0^2/a$, characteristic time; $Pr = \nu/a$, Prandtl number; $Gr = q\beta(T_0 - T_C)\tilde{x}_0^3/\nu^2$, Grashof number.

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UNSTEADY CONVECTIVE HEAT TRANSFER IN POTENTIAL FLOW

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Analytical relations are obtained for the unsteady temperature field in potential flow over a flat plate and a cylinder.

Works [1-3] have examined stationary forced convection in potential flow of a liquid over bodies.

The present paper determines the unsteady temperature fields in longitudinal flow over a flat plate with boundary conditions of the first and second kinds, with and without allowance for thermal radiation and motion of the cylinder in the flow with boundary conditions of the first kind. This type of problem is described by the equations for the fluid temperature T, the velocity potential φ, and the pressure p:

$$\Delta\varphi = 0, \tag{1}$$

$$\frac{\partial T}{\partial t} + (\operatorname{grad} T,\ \operatorname{grad}\varphi) = a\Delta T, \tag{2}$$

$$p = p_0 - 0.5\rho(\operatorname{grad}\varphi)^2 - \rho\frac{\partial\varphi}{\partial t} - \rho(\vec{g},\ \vec{r}) \tag{3}$$

and initial and boundary conditions on the body surface

$$\frac{\partial\varphi}{\partial n} = U, \tag{4}$$

$$T(\vec{x},\ 0) = T_0(\vec{x}),\ T|_{\vec{x}\in S} = f(\vec{x}_s,\ t),\ \lambda\frac{\partial T}{\partial n}\Big|_{\vec{x}\in S} = g(\vec{x}_s,\ t),$$
$$\lambda\frac{\partial T}{\partial n}\Big|_{\vec{x}\in S} = \sigma(T^4|_{\vec{x}\in S} - T_\infty^4),\ T|_{\vec{x}\to\infty} = T_\infty. \tag{5}$$

We assume that the thermophysical properties of the liquid are independent of temperature and pressure and that a similarity transformation $x_i = \sqrt{a}\,y_i$ has been derived which results in the coefficient of ΔT reducing

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to 1. Work [4] has shown that the problem (1), (4) has a solution which exists and is unique. We shall seek the solution of the first boundary-value problem, (2), (5) in the class of functions $W_{2,\exp(-\varphi)}^{1,1}(\Omega_t)$ [5]. We introduce ψ_0, a function of the spatial coordinate $\vec{x}$ and the time t, satisfying the conditions

$$\psi_0(\vec{x},\ 0)=0,\ \psi_0(\vec{x},\ t)|_{\vec{x}\in S}=f(\vec{x}_s,\ t),$$

$$F=\left[\exp(-\varphi)\frac{\partial\psi_0}{\partial t}-\operatorname{div}(\exp(-\varphi)\operatorname{grad}\psi_0)\right]\in L_2(\Omega_t),$$
$$F_t\in L_2(\Omega_t).$$

We first consider the internal problem for Eqs. (2) and (5). A generalized solution of this problem is given by the function $T=\psi_0+\vartheta$, where ϑ belongs to $\overset{\circ}{W}{}_{2,\exp(-\varphi)}^{1,1}(\Omega_t)$ and satisfies the integral identity

$$\int_0^t\int_\Omega\left(\exp(-\varphi)\frac{\partial\vartheta}{\partial t}A-FA-\exp(-\varphi)\sum_{i=1}^{3}\frac{\partial\vartheta}{\partial x_i}\cdot\frac{\partial A}{\partial x_i}\right)dxdt=0 \tag{6}$$

for any function $A\in\overset{\circ}{W}{}_{2,\exp(-\varphi)}^{1,1}(\Omega_t)$. We shall show that the function ϑ is unique. Let ϑ_1, ϑ_2 be two generalized solutions of Eqs. (2) and (5); then, introducing the notation

$$A=\begin{cases}\omega=\vartheta_2-\vartheta_1, & 0\leqslant t<s,\\ 0, & s<t<s_0,\end{cases}$$

and using Eq. (6), following transformations we obtain

$$\int_0^s\int_\Omega\exp(-\varphi)\left[\omega^2\frac{\partial\varphi}{\partial t}+\sum_{i=1}^{3}\left(\frac{\partial\omega}{\partial x_i}\right)^2\right]dxdt+\int_\Omega\exp(-\varphi)\,\omega^2dx=0. \tag{7}$$

If φ is independent of time, from Eq. (7) we obtain $\omega=0$, $\vartheta_1=\vartheta_2$. If φ is a function of time, then, from the condition that the pressure and liquid velocity are finite, using the Bernoulli integral (3), we obtain the result that $|\partial\varphi/\partial t|<K_0$. Then, using the fact that $\partial\varphi/\partial t$ is finite, applying the Sobolev imbedding theorems for the space $\overset{\circ}{W}{}_2^{1,1}(\Omega_t)$ [4], and introducing the notation $K=\int_\Omega\exp(-\varphi)\omega^2dx$, we find that K satisfies the differential inequality

$$\frac{dK}{dt}\leqslant\alpha K,\ K(0)=0.$$

Hence it follows that $K=0$.

We now consider the external problem (2), (5). We postulate that at infinity $\varphi\sim1/r$, $\vartheta\sim r^{-(1.5+\varepsilon)}$, $\partial\vartheta/\partial x_i\sim r^{-2.5}$ and write Eq. (7) for the region $\Omega=\Omega_R$, where Ω_R is a sphere of radius R containing S; we let $R\to\infty$, apply the same arguments for ϑ as in considering the internal problem, and obtain the result that ϑ is unique.

Replacing the variables x and y by φ and ψ, and T by $\Theta=(T-T_\infty\exp(-\varphi/2a)$ in Eqs. (2) and (5), we obtain the following equation for Θ:

$$\frac{\partial\Theta}{\partial t}=Aa\left(\frac{\partial^2\Theta}{\partial\varphi^2}+\frac{\partial^2\Theta}{\partial\psi^2}-\frac{\Theta}{4a^2}\right) \tag{8}$$

and the initial condition

$$\Theta|_{t=0}=\exp\left(-\frac{\varphi}{2a}\right)(T_0(\vec{x})-T_\infty)=B_0. \tag{9}$$

For the first boundary-value problem the boundary condition is

$$\Theta|_{\vec{x}\in S}=(f(\vec{x}_s,\ t)-T_\infty)\exp\left(-\frac{\varphi}{2a}\right)=f_1. \tag{10}$$

We shall solve this for longitudinal flow over a flat plate with velocity U.

Applying a Laplace transform in the variable t to Eqs. (8), (9), and (10) and a Fourier transform in φ, we obtain

$$\Theta = p\bar{B}_0\left(p + \alpha_2^2 aU^2 + \frac{U^2}{4a}\right)^{-1} + \left(\bar{f}_1 - p\bar{B}_0\left(p + \alpha_2^2 U^2 a + \frac{U^2}{4a}\right)^{-1}\right)$$
$$\times \exp\left(-\frac{\psi}{U\sqrt{a}}\sqrt{p + \alpha_2^2 U^2 a + \frac{U^2}{4a}}\right). \tag{11}$$

Here

$$\bar{\Theta} = p\int_0^\infty \int_{-\infty}^\infty \Theta \exp(-pt + i\alpha_2\varphi)\, dt d\varphi, \quad \bar{B}_0 = \int_{-\infty}^\infty B_0 \exp(i\alpha_2\varphi)\, d\varphi,$$

$$F_1 = \int_{-\infty}^\infty f_1 \exp(i\alpha_2\varphi)\, d\varphi, \quad \bar{f}_1 = p\int_0^\infty F_1 \exp(-pt)\, dt.$$

Transferring from the transform space to the original in Eq. (11), and using data from [6], we obtain a temperature distribution in the form

$$T = T_\infty + \exp\left(\frac{\varphi}{2a}\right)\frac{1}{2\pi}\left[\int_{-\infty}^\infty \bar{B}_0 \exp\left(-\left(\frac{U^2}{4a} + \alpha_2^2 U^2 a\right)t - i\varphi\alpha_2\right) d\alpha_2\right.$$
$$- \int_{-\infty}^\infty \bar{B}_0 \operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}}\right)\exp\left(-t\left(\alpha_2^2 U^2 a + \frac{U^2}{4a}\right) - i\varphi\alpha_2\right) d\alpha_2$$
$$\left. + 0.5\frac{\partial}{\partial t}\int_{-\infty}^\infty \int_0^t F_1(t-\tau, \alpha_2) K_1(\alpha_2, \psi, \tau)\exp(-i\varphi\alpha_2)\, d\alpha_2 d\tau\right], \tag{12}$$

$$K_1 = \exp\left(-\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}}\right.$$
$$\left. - U\sqrt{a}\sqrt{\left(\alpha_2^2 + \frac{1}{4a^2}\right)t}\right) + \operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}}\right.$$
$$\left. + U\sqrt{a}\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\exp\left(\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right).$$

Here $\varphi = Ux$, $\psi = Uy$. The local heat-transfer coefficient α_φ is calculated from the relation

$$\alpha_\varphi = \frac{\lambda U}{2\pi f}\exp\left(\frac{\varphi}{2a}\right)\left[\frac{\partial}{\partial t}\cdot\frac{1}{\sqrt{\pi t}}\int_{-\infty}^\infty \bar{B}_0 \exp\left(-t\left(\alpha_2^2 U^2 a + \frac{U^2}{4a}\right)\right.\right.$$
$$\left.\left. - i\varphi\alpha_2\right) d\alpha_2 - \frac{\partial}{\partial t}\int_{-\infty}^\infty \int_0^t F_1(t-\tau, \alpha_2) K_0(\tau, \psi, \alpha_2)\, d\alpha_2 d\tau\right],$$

$$K_0 = \left(\sqrt{\alpha_2^2 U^2 a + \frac{U^2}{4a}}\operatorname{erfc}\left(\sqrt{t\left(\alpha_2^2 U^2 a + \frac{U^2}{4a}\right)}\right) + \frac{1}{\sqrt{\pi t}}\exp\left(t\left(\alpha_2^2 U^2 a + \frac{U^2}{4a}\right)\right)\right)\exp(-i\alpha_2\varphi).$$

For the second boundary-value problem the boundary condition is

$$\left.\frac{\partial\Theta}{\partial\psi}\right|_{\psi=0} = \frac{q(\varphi U^{-1}, t)}{\lambda U}\exp\left(-\frac{\varphi}{2a}\right) = q_1.$$

Solving this for longitudinal flow over a flat plate with velocity U, using the same method as in the first boundary-value problem, we obtain the following expressions for the temperature distribution in the liquid and the local heat-transfer coefficient:

$$T = T_\infty + \frac{1}{2\pi}\exp\left(\frac{\varphi}{2a}\right)\left[\int_{-\infty}^\infty \bar{B}_0 \exp\left(-\frac{U^2 t}{4a} - i\varphi\alpha_2 - \alpha_2^2 U^2 at\right) d\alpha_2 - \frac{\partial}{\partial t}\int_{-\infty}^\infty \int_0^t Q_1(t-\tau, \alpha_2) K(\tau, \psi, \alpha_2)\, d\alpha_2 d\tau\right],$$

$$\alpha_\varphi = \frac{2\pi}{\Delta T_0} q \exp\left(\frac{\varphi}{2a}\right), \tag{13}$$

where

$$Q_1 = \int_{-\infty}^{\infty} q_1 \exp(i\alpha_2\varphi)\, d\varphi, \quad K = \exp(-i\varphi\alpha_2)\left\{\exp\left(-\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\right.$$

$$\times\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)^{-1/2} - 0.5\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)^{-1/2}\left[\exp\left(\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\right.$$

$$\left.\left.\times \operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}} + \sqrt{t\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)}\right) + K_1\right]\right\},$$

$$\Delta T_0 = \frac{\partial}{\partial t}\int_{-\infty}^{\infty}\int_0^t Q_1(t-\tau, \alpha_2)\, K_2(\tau, \alpha_2, \varphi)\, d\alpha_2 d\tau + \int_{-\infty}^{\infty} \bar{B}_0 \exp\left(-i\varphi\alpha_2 - \frac{U^2t}{4a} - \alpha_2^2U^2at\right) d\alpha_2\Big\},$$

$$K_1 = \exp\left(-\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\operatorname{erfc}\left(\sqrt{\tau\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)} - \frac{\psi}{2U\sqrt{a\tau}}\right),$$

$$K_2 = \left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)^{-1}\exp(-i\varphi\alpha_2)\operatorname{erfc}\left(\tau\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)\right).$$

We find the temperature distribution in the liquid in longitudinal flow over a plate with velocity U for the condition that for $t > 0$ the heat flux distribution $q(\vec{x}, t)$ on the plate is known and there is radiation and absorption of heat in the surrounding space. With the assumption that $|(T-T_\infty)/T_\infty|_{y=0} \ll 1$ the boundary condition transforms to the following form:

$$\frac{\partial\Theta}{\partial\psi} - N\Theta|_{\psi=0} = q_1. \tag{14}$$

By solving the initial-value problems (8), (9), (14) in the same way as for the first boundary-value problem, we obtain the following formula for the temperature distribution in the liquid:

$$T = T_\infty - \frac{1}{2\pi}\exp\left(\frac{\varphi}{2a}\right)\left[\frac{\partial}{\partial t}\int_0^t\int_{-\infty}^{\infty} Q_1(t-\tau, \alpha_2)\, K_1(\tau, \psi, \alpha_2)\, d\alpha_2 d\tau\right.$$

$$\left. - \int_{-\infty}^{\infty} \bar{B}_0 \exp\left(-\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)t\right) K_2(t, \psi, \alpha_2)\, d\alpha_2\right].$$

Here

$$K_1 = \frac{\exp(i\alpha_2\varphi)}{2} U\sqrt{a}\left[\exp\left(-\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\left(NU\sqrt{a}\right.\right.$$

$$\left. + \sqrt{\alpha_2^2U^2a + \frac{U^2}{4a}}\right)^{-1}\operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}} - \sqrt{\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)t}\right)$$

$$+ \exp\left(\psi\sqrt{\alpha_2^2 + \frac{1}{4a^2}}\right)\left(NU\sqrt{a} - \sqrt{\alpha_2^2U^2a + \frac{U^2}{4a}}\right)^{-1}$$

$$\times \operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}} + \sqrt{\left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)t}\right) - 2\operatorname{erf}\left(\frac{\psi}{2U\sqrt{at}} + NU\sqrt{at}\right)$$

$$\left.\times\left(N^2U^2a - \alpha_2^2U^2a - \frac{U^2}{4a}\right)^{-1}\exp\left(N\psi - N^2U^2at - \left(\alpha_2^2U^2a + \frac{U^2}{4a}\right)t\right)\right],$$

$$K_2 = \left(1 - \frac{1}{U\sqrt{a}}\left[\operatorname{erf}\left(\frac{\psi}{2U\sqrt{at}}\right) - \exp(N\psi + NU^2at)\operatorname{erfc}\left(\frac{\psi}{2U\sqrt{at}} + NU\sqrt{at}\right)\right]\right)\exp(i\alpha_2\varphi).$$

We now consider the motion of an infinite cylinder with velocity U in potential flow of a liquid. Let the initial liquid temperature be $T_0(r, \alpha)$, and for $t > 0$ let the temperature distribution $f(\alpha, t)$ be given on the cylinder surface. The liquid temperature far from the body is T_∞.

With the assumption of potential flow the stream function ψ and the velocity potential φ have the form

$$\varphi = -\frac{UR^2}{r}\cos\alpha,\quad \psi = \frac{UR^2}{r}\sin\alpha.$$

This initial-value problem for the temperature distribution in the liquid, following the transformations

$$\varphi = \frac{\rho}{UR}\cos\beta,\quad \psi = \frac{\rho}{UR}\sin\beta,\ \mathrm{Fo} = \frac{at}{R^2}$$

reduces to the form

$$\frac{\partial\Theta}{\partial\,\mathrm{Fo}} = \rho^4\left(\frac{\partial^2\Theta}{\partial\rho^2} + \frac{1}{\rho^2}\cdot\frac{\partial^2\Theta}{\partial\beta^2} + \mathrm{Pe}^2\Theta + \frac{1}{\rho}\cdot\frac{\partial\Theta}{\partial\rho}\right),$$

$$\Theta|_{\mathrm{Fo}=0} = (T_0 - T_\infty)\exp\left(-\frac{\varphi}{2a}\right) = Q_0(\beta,\ \rho), \tag{15}$$

$$\Theta|_{\rho=1} = (f - T_\infty)\exp\left(-\frac{\varphi}{2a}\right) = g(\beta,\ \mathrm{Fo}).$$

We seek a solution of Eq. (15) in the form

$$\Theta = \sum_{n,m=0}^{\infty}\mathrm{Pe}^{2m}(A_{n,m}(\mathrm{Fo},\ \rho)\cos n\beta + B_{n,m}(\mathrm{Fo},\ \rho)\sin n\beta),$$

$$g = \sum_{n=0}^{\infty}(g_{0,n}\cos n\beta + g_{1,n}\sin n\beta), \tag{16}$$

$$Q_0 = \sum_{n=0}^{\infty}(Q_{0,n}\cos n\beta + Q_{1,n}\sin n\beta).$$

Substituting Eq. (16) into Eq. (15), we obtain the result that the functions $A_{n,m}$ are solutions of the following problem:

$$\frac{\partial A_{n,0}}{\partial\,\mathrm{Fo}} = \rho^4\left(\frac{\partial^2 A_{n,0}}{\partial\rho^2} + \frac{1}{\rho}\cdot\frac{\partial A_{n,0}}{\partial\rho} - \frac{n^2}{\rho^2}A_{n,0}\right),$$

$$\frac{\partial A_{n,m+1}}{\partial\,\mathrm{Fo}} = \rho^4\left(\frac{\partial^2 A_{n,m+1}}{\partial\rho^2} + \frac{1}{\rho}\cdot\frac{\partial A_{n,m+1}}{\partial\rho} - A_{n,m} - \frac{n^2}{\rho^2}A_{n,m+1}\right), \tag{17}$$

$$A_{n,0}|_{\mathrm{Fo}=0} = Q_{0,n},\ A_{n,0}|_{\rho=1} = g_{0,n},\ A_{n,m+1}|_{\mathrm{Fo}=0} = 0,\ A_{n,m+1}|_{\rho=1} = 0.$$

The functions $B_{n,m}$ satisfy this type of initial-value problem. The solutions of Eq. (17) can be found in the form

$$A_{n,m} = \sum_{k=0}^{\infty}A_{n,m}^k(\mathrm{Fo})J_n\left(\frac{\lambda_{n,k}}{\rho}\right),\quad B_{n,m} = \sum_{k=0}^{\infty}B_{n,m}^k(\mathrm{Fo})J_n\left(\frac{\lambda_{nk}}{\rho}\right), \tag{18}$$

where λ_{nk} are roots of the equation

$$J_n(\lambda_{nk}) = 0.$$

It can be shown that the functions $J_n(\lambda_{n,k}/\rho)$ are orthogonal in $[1,\ \infty)$ with weight $1/\rho^3$; by substituting Eq. (18) into Eq. (17) and using the orthogonality of $J_n(\lambda_{n,k}/\rho)$, we obtain the result that $A_{n,m}^k$ and $B_{n,m}^k$ satisfy the ordinary linear differential equations of the first order. By using the method of variation of an arbitrary constant to solve these equations, we find $A_{n,m}^k$ and $B_{n,m}^k$. Substituting $A_{n,m}^k$ and $B_{n,m}^k$ into Eqs. (18) and (16), we find the temperature distribution in the liquid, which is not presented here because the expressions are very cumbersome.

NOTATION

$\vec{x}$, three-dimensional coordinate; t, time; r, α, polar coordinates; x, y, Cartesian coordinates: the x axis is along the plate and the y axis is perpendicular to the plate; φ, velocity potential; $A = (\mathrm{grad}\,\varphi)^2$; ψ, stream function; T, p, the liquid temperature and pressure; T_0, initial liquid temperature; T_∞, p_0, the temperature and pressure far from the body; n, normal to the body; S, the body surface; U, the flow velocity; f, initial temperature distribution on the body;

q, heat flux density; Ω, region occupied by the liquid; $\vec{r}$, distance from an arbitrary point in the liquid to the body; $\Omega_t = \Omega \times [0, t]$; g, acceleration due to gravity; ρ, λ, a, the liquid density, thermal conductivity, and thermal diffusivity; σ, the integral emissivity; $N = 4\sigma T_\infty^3/\lambda U$; $L_2(\Omega_t)$, Hilbert space; $\overset{\circ}{W}{}_2^{1,1}(\Omega_t)$, $\overset{\circ}{W}{}_{2;\exp(-\varphi)}^{1;1}(\Omega_t)$, Sobolev space.

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TWO-DIMENSIONAL TEMPERATURE DISTRIBUTION IN A CERAMIC-BASED ELECTRODE

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A study has been made of the thermal processes in the electrode units in an MHD channel; generalized relationships between the geometrical parameters of the blocks and the parameters of the working body have been derived.

Much attention is now being given to large MHD systems containing sectional ceramic electrodes for use in fully commercial or pilot MHD stations [1]. The viability and working lives of such systems are largely determined by the thermal conditions in the electrode blocks.

There are several papers on the temperature distributions in such blocks; for instance, temperature distributions have been determined [1, 2] for ceramic electrode modules enclosed in metal matrices. Estimates have been made [1] of the maximum temperature in a module and the time needed to reach the steady thermal state for blocks of various sizes and various heat-flux levels at the MHD channel wall.

However, most studies [1-5] are based on solving the thermal-conduction equations subject to major simplifications (constant temperature in the metal matrix, constant thermophysical parameters of the electrode materials, etc.), which substantially restrict the applicability of the results to viability evaluation.

§1. Figure 1a, b shows some typical electrode schemes based on ceramic modules made of zirconium dioxide ZrO_2 [2, 3]. A ceramic module is enclosed in a metal cooling matrix, while the electrical insulation is provided by plates of Al_2O_3 or MgO.

The metal matrix in Fig. 1a performs two functions: it cools the ceramic element and also handles the current through the upper parts of the metal edges. Since ZrO_2 ceramic is of fairly high electrical conductivity ($\sigma >$ 10-20 mho/m) only at high temperatures ($T \geqslant 1100$-1200°K) [1], the edge of the matrix must be made of heat-resisting steel.

In Fig. 1b, the current is carried by high-temperature metal grid or plate embedded in the ceramic element, which reduces the severity of the working conditions for the metal cooling edges and allows one to make the matrix of a metal of high thermal conductivity such as copper.

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